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The Report of the Gravity
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Godderd Space Flight Center Greenbelt, Maryland 20771



REPORT OF THE GRAVITY FIELD WORKSHOP

Foreword

Determining the earth's gravity field through observations of near-earth satellites has been a long-standing NASA goal. Recent investigations indicate that the proposed gravity mapping satellite, GRAVSAT, which is planned for launch by 1988, will greatly improve our knowledge of the earth's gravity field. However, there is a significant need for an interim gravity field 1) to satisfy current requirements for geodetic, geophysical and oceanographic data analysis and 2) to develop an adequate basis for initiating the analysis of the data collected during the GRAVSAT mission. Consequently, a Gravity Field Workshop was convened at the NASA Goddard Space Flight Center to provide recommendations on the structure of a program to determine such a gravity field. The conclusions reached during this workshop are discussed in this report.

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1. Introduction

The requirements for an accurate representation of the earth's gravity field are broad [1]-[6]. The spacific needs span a wide range of scientific and engineering applications [1]. Early in the space program, it was recognized that combining accurate observations of near-earth satellite motions with accurate terrestrial gravity measurements provided a unique approach for determining the global gravity field.

Consequently, for almost two decades the scientific community has employed artificial atellites to establish a global geopotential model for the earth's gravity field. Significant improvement in the knowledge of the earth's gravity has been achieved through this approach. At present, the uncertainty in the spherical harmonic coefficients up to degree and order 40 for the current gravity models yields an rms geoid height error of approximately 70 cm. Despite this improved knowledge, further progress in many geodetic and geophysical disciplines is now limited by inaccuracies in the gravity field model. There is a need for an improved gravity field model to enable the determination of more accurate satellite orbits to fully utilize certain satellite-determined data for scientific applications. For example, there is a mismatch currently between the orbital accuracy for Seasat (about 1 m) and the precision of the altimeter measurement (about 5 cm). In addition, precise satellite orbits are required to support a number of navigation and precise point positioning applications, including global surveys and tectonic plate motion. The gravity field also determines the shape of the good which is used as the basis for applying altimeter measurements in oceanographic and geophysical investigations; however, the present accuracy of the geoid is inadequate for interpreting contemporary altimeter measurements. Uncertainty in the geopotential model is the primary limitation in achieving successful results in a number of these applications.

As a consequence of these facts, a Gravity Field Workshop was convened at the NASA Goddard Space Flight Center to review the actions which could be taken prior to a GRAVSAT mission to improve the earth's gravity field model. This review focused on the potential improvements in the earth's gravity field which could be obtained using the current satellite and surface gravity data base. In particular, actions to improve the quality of the gravity field determination through refined measurement corrections, selected data augmentation and a more accurate reprocessing of the data were considered. In addition, recommendations were formulated which define actions which NASA should take to develop the necessary theoretical and computation techniques for gravity model determination and to use these approaches to improve the accuracy of the earth's gravity model.

2. Plans for GRAVSAT

For more than a decade, consideration has been given to improving measurements of the earth's gravity field through a global gravity satellite (GRAVSAT) mapping misson. In 1979, a National Research Council panel formed specific recommendations for such a mission [2]. These sessions led to the GRAVSAT Users Working Group who defined the requirements for the gravity field determination and assessed the mission's feasibility [3] [4].

The proposed GRAVSAT mission would measure the range rate between two satellites at a height of 160 km with a precision of \pm 1 μ m/s. Previous studies have indicated that global measurements of this accuracy would allow determination of global equivalent $1^{\circ}x1^{\circ}$ mean gravity anomalies to \pm 2 mgals and an equivalent $1^{\circ}x1^{\circ}$ mean geoid undulation to \pm 4 cm [7]. Accuracy of the geoid heights from GRAVSAT for the spherical harmonic degrees from 2 to 180 was estimated to be \pm 0.3 cm [6] provided long wavelength error sources do not contaminate the results. Such information represents a factor of 20 to 200 improvement over current knowledge of the gravity field.

Although appropriate hardware and data analysis procedures are now being developed, GRAVSAT is not currently an approved NASA mission. If it is approved, launch will not be anticipated prior to 1988, with full results becoming available during the early 1990 time period. Hence, to meet the demands of current geophysical and oceanographic programs, an interim gravity field program is required. In addition, advantage should be taken of this lead time to develop appropriate analytical techniques for the analysis of the GRAVSAT data (see Section 5).

3. Need for Near-Term Gravity Field Model Improvements

About half of the rms geoid error of 70 cm for coefficients 40 as cited in Section 1 is due to uncertainties in the spherical harmonic coefficients below degree 10. Reprocessing previously collected data with a refined model, combined with improved analysis techniques which now exist and the addition of selected new data should reduce this portion of the error spectrum by as much as 50%. In addition, improved reliability of the estimate of the error in the computed geoid will result. A program to achieve these improvements is essential to satisfying oceanography requirements if the GRAVSAT misson is not undertaken at a reasonably early date. In addition, these contributions could serve as checks on the results obtained from the GRAVSAT mission.

Analysis of the tracking data from Lageos, Starlette and Beacon Explorer-C to determine crustal motions (both regional and global), polar motion and variations in the earth's rotation rate may also be influenced by these long wave length gravity model errors. In addition, mantle convection studies on a global scale will ultimately reach the level where they will require a more accurate definition of the long wave length features of the earth's gravity field.

Currently, ongoing geodynamic investigations interpret features of the oceanic geoid as resulting from the aging of the lithosphere, its interaction with hot spots, subduction, etc. These studies depend on short wave length features of the gravity field, and the accuracy of the current altimetric geoid is adequate for a number of these studies, but some future investigations will require improvements in the accuracy of the short wave length features. In addition, the error sources inherent in these geoid models must be described more precisely for coordinated regional studies which are currently being performed using bathymetry, sediment thickness, heat flow and other data.

The previous paragraphs indicate a specific need for improvement in the existing gravity field for both the long and short wave length features. Consequently, we have defined the goal for the gravity field improvement as a 50% reduction in the long wave length errors (i.e. below degree and order 10), along with a corresponding improvement for the shorter wave lengths.

The primary focus of this report is directed to 1) the near-term actions required to obtain interim improvements in the current gravity model and 2) the long-term actions which should be initiated to provide a basis with which to achieve the objectives of the GRAVSAT mission.

The discussion of gravity interpretation in the next section is limited to the requirements for the near-term gravity model improvement. A more general discussion in Section 5 concerns long-term improvements.

4. Near-Term Improvements

A. Introduction

Significant improvements can be made in both the accuracy of the numerical values of the coefficients in the current gravity field models and the uncertainty associated with these estimates. The improvements can be obtained through two significant actions: 1) use of improved physical, mathematical and statistical models for computing the gravity field model and 2) the incorporation of

additional observational data. These actions would improve significantly both the solution and the reliability of the error estimates for the gravity field model. Paragraphs C through E below discuss areas of model improvement which should be addressed prior to attempting to develop a new gravity model solution.

As stated in the previous discussion, the primary objective of the gravity model improvement should be focused to obtain a 50% improvement in the accuracy of the long wave length features of the gravity field. This objective can be achieved by:

- Reprocessing a selected set of the available laser, S-band, doppler and satellite-to-satellite tracking data using improved models for measurement corrections and for the satellite's motion:
- 2. Incorporating information derived from the analysis of the motion of synchronous satellites and other satellites which are in resonant orbits, i.e., by use of available mean elements from a variety of satellites to extract information for coefficients of 10th to 30th order by analysis of resonance effects:
- Processing a selected subset of DOD-acquired doppler data from four satellites with different orbital inclinations.
- 4. Acquiring and processing laser observations from four selected satellites in a new tracking campaign,
- 5. Reprocessing Baker-Nunn and other available optical data, and
- Incorporating information on the zonal gravity coefficients as derived from a variety of satellites.

With additional effort, our knowledge of the gravity field could be improved at wave lengths down to about 200 km, i.e., degree and order 200. This additional effort entails processing additional gravimetry data and altimeter measurements made from GEOS-3, Seasat and GEOSAT.

B. Data Collection

a) Satellite Data

The tracking data types which should be included in the determination of an improved gravity field model are: Laser, camera, electronic ground-based systems (DOD Transit Doppler, S-Band), electronic satellite-to-satellite tracking (ATS-6/GEOS-3, TDRSS and

GPS), satellite altimeter (GEOS-3, Seasat and GEOSAT), and selected mean orbital element sets obtained from current operational satellite programs, e.g., NAVSPASUR, etc. Only about 30% of the evailable data have been used in any single determination of the gravity field model.

The Transit doppler data, required for the improved gravity model, consist of four weeks of existing observations of the NOVA, Seasat, GEOS-3 and one satellite near the critical inclination. Doppler data acquired during the MERIT main campaign should be exploited also. The existing laser ranging data from Lageos and Starlette should be supplemented by additional satellite data acquired from high-precision laser tracking stations deployed at widely dispersed geographic sites. In addition to data from Lageos and Starlette, data should be obtained by tracking the existing satellites, Beacon Explorer-C, GEOS-1, GEOS-2 and GEOS-3, for appropriate lengths of time.

Other independently derived parameters can be incorporated into the data set. For example, baseline lengths and directions derived from VLBI measurements will improve the solution for station locations. Station coordinates can be constrained also through the use of surface surveys. Finally, polar notion and UT1 solutions from the VLBI Polaris Project, BIH and laser ranging will lead to improved processing of the satellite tracking data.

b) Terrestrial Data

Surface gravity data contribute materially to the accuracy of earth gravitational field models. Most current grayity field determinations incorporate 5° equal area blocks of 1°x1° mean free-air anomalies. The contribution of such data can be increased substantially by correcting several deficiencies in the current data. In addition, efforts to improve the collection and assessment of the 1°x1° terrestrial gravity field data should be intensified where possible. Organizations such as the Defense Mapping Agency, who have an ongoing gravity collection program, should be requested to acclerate these activities to produce new anomaly data for use in gravity modeling.

Special consideration should be given to estimating gravity anomalies in areas where no direct observations are available. Currently, certain land areas, such as parts of South America, Africa, Asia, etc., have anomaly estimates based on geophysical correlation techniques which may be quite unreliable. Improvement of such anomaly estimates is urgently needed.

C. Satellite Modeling

a) Physical Models

Today, the phenomena affecting the data types used to determine the gravity field are better understood. As a result, better models can be used to correct for the effects of atmospheric drag (including both atmospheric density and projected area variations); tropospheric and ionospheric refraction; direct and indirect radiation pressure; land, ocean and atmospheric tides; sea—surface topographic corrections; and special and general relativity. Furthermore, the use of existing gravity fields should improve the assumption of linearity and provide more accurate variational equations.

b) Mathematical Models

Recent improvements in estimation and computer technology (see Section E and F) should be used in the computation of an improved gravity field solution. For example, to ensure the numerical stability of the gravity field solution, there is a need to investigate the accuracy of currently used numerical approximation techniques, such as the use of analytical partial derivatives, and the methods used to determine the gravity field estimate. In particular, the use of orthogonal transformation techniques for solving the linear system of observation—state equations should be considered. To reduce the computer time required for processing individual measurements, techniques for data compression should be used. The generation of a single set of compressed data from the raw data sets should be developed for all gravity field modeling efforts.

c) Statistical Modeling

Better statistical models will contribute both to an improved gravity field model and an improved estimate of its accuracy. Areas to be improved include: 1) removal of correlated error in altimeter data due to orbit error and 2) improvements in the representation of the correlated error in sea surface topography. Errors in the satellite ephemeris due to truncation of the gravity field also produce systematic effects which need to be modeled properly.

D. Parameter Estimation

a) Reprocessing of Data

An improved gravity field solution can be obtained by reprocessing old data with the improved physical and mathematical modeling described above. Improved weighting of the data and the estimation of a more complete gravity field model will yield accuracy

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improvements. Realistic error estimates can be obtained also through more complete modeling of non-gravitational parameters and better statistical estimates for the unmodeled effects. The compressed data equations resulting from these efforts would be appropriate for combination with new data.

b) Data Not Previously Processed

With the inclusion of new data and a more accurate reduction of the existing data, the precision and accuracy of the geopotential will be improved. The observability of the terms defined in the current models will be enhanced through the better global data distribution. Gains in the model accuracy will occur also from the increased accuracy of the new data. As a consequence, there will be reduced correlation and reduced errors due to unmodeled perturbations. To ensure that the error model for the field can be interpreted statistically, a consistent degree and order for the parametric representation of the gravity field model should be used for the reductions of all data sets.

c) Solution Validation

The utility or acceptance of a gravity field model by the scientific community is dependent on its accuracy and its perceived validity. Consequently, a study should be initiated to identify, for various contemporary applications, a set of standards, techniques and/or procedures that can be used to confirm a given model's accuracy. These procedures should be used to evaluate the accuracy of interim gravity field solutions.

E. New Estimation Methods for Combination Solutions

To improve the accuracy of the high-resolution components of the gravity field, i.e., wave lengths shorter than 1000 km, solutions which combine satellite and terrestrial data are needed. Near-term improvements of gravity models can be obtained only by combining data from many techniques, e.g., ground-based satellite tracking, satellite altimeter, satellite-to-satellite tracking, gravimeters, gravity gradiometers, astrogeodetic and inertial surveying. The combination of different ground-based satellite tracking data types has been used as a standard technique for years, but some combined satellite-terrestrial solutions are based on techniques that do not make full use of terrestrial data information, e.g., the surface data are compressed to 5° mean values. Algorithms are needed which produce broadband, i.e., to wavelengths less than 200 km, gravity models based upon large volumes of data obtained from sensors of diverse measurement characteristics and accuracy.

An algorithm for combining data from such diverse measurements requires a parametric model for the gravity field, models for the sensor measurement errors and a description of the survey geometry. Examples of models that might be considered are spherical and ellipsoidal harmonics, Fourier series and local formulations, such as sampling functions, point masses, finite elements, etc. Consideration must be given to non-isotropic, non-stationary and non-Gaussian statistical descriptions in both global and regional formulations. The sensor error models should include both instrumental and environmental effects and should account for measurement error correlations. The spatial distribution of the measurements should be recognized in the estimation algorithms.

Significant new efforts are required to develop two-dimensional (planar and spherical) estimation algorithms. Algorithms are needed to combine low-resolution global data with high-resolution terrestrial data collected by different sensors in limited regions. Furthermore, the algorithms must retain the resolution afforded by the available terrestrial data and provide realistic measures of accuracy for the estimated gravity field.

Consideration must also be given to numerical conditioning and to tradeoffs between algorithm accuracy and required computation resources (see Section F).

F. Implications of Computer Technology Advances

Advances in computer architecture and physical memory can be exploited in applying improved numerical techniques to the large parameter gravity field estimation problem. Consequently, numerical algorithms should be developed for integration of the satellite equations of motion and solution of large systems of linear equations that use these improved estimation and combination techniques. Furthermore, large system analyses should benefit from current and projected advances in computational performance, which result from the use of modern computer architecture. Recent developments which may be appropriate for the gravity field determination include parallel and vector processors, Very Large Scale Integration and other anticipated architectural developments.

5. Long-Term Improvements

The new global measurement systems anticipated within the subsequent decade include high-accuracy satellite-to-satellite rangerate and, later, satellite gradiometry. These measurements will differ from the data used for current gravity field analysis based on satellite altimetry, since they measure the gravity field globally at the spacecraft altitude. These data types will make possible both a much more detailed description of the gravity field and complete global coverage.

The enhanced detail associated with the global coverage will necessitate an attendant increase in the data quantity. The sixmonth GRAVSAT mission should generate approximately 3.5 million individual measurements of the gravity field at satellite altitude. However, the global coverage should allow the use of more economical data analysis techniques, such as Fourier analysis or local smoothing and interpolation, rather than the conventional least squares inversion. Nevetheless, the global methods are subject to systematic error and require orbit improvement to realize their full capability.

Orbit improvement, which entails iterative analysis of globally distributed satellite tracking, requires a reasonably accurate initial gravity field. The possibility of systematic error, particularly east-west variations of the field which depend on orbit-to-orbit differences, suggests that an independent solution is desirable to ascertain the existence of such systematic error effects. In addition, complete utility of the data at the earth's surface will require that the problems associated with the downward continuation of the data be resolved. The eventual option solution may combine the classical data (for the longer wave length variations) with new data gathered during a GRAVSAT mission.

In addition, geophysical studies will require more accurate gravity fields, as well as better error models and mathematical representations. Gravity anomalies result from the combined attraction of density irregularities which in turn cause stress. In tectonic and convection studies, gravity data are used in combination with plate velocities, heat flow, reismic data, bathymetry, etc. However, as models become refined, the shorter wave length features of the gravity field become key diagnostics. For example, such features measured by the Apollo 15 orbiter helped identify the lunar mascons as largely shallow slabs. On the example, the stress state of a subducted slab has been inferred from the oceanward positive swell in the geoid.

Regional isostatic compensation is an important clue to lithospheric stress and the mechanical properties of the upper mantle. A more accurate higher order gravity field would help resolve the depth of inferred mass anomalies, a major unknown in the mechanism of isostatic compensation. By constraining models of plate tectonics and convection, gravity data will also help answer questions about the separation between upper and lower mantle flows.

The foregoing discussion indicates that, in addition to the requirements for accurate long wave gravity field features, there is a need to determine the gravity field to as high a degree and order as possible through the use of existing data and new analysis techniques. Both activities will foster scientific and technical interest in the gravity field and, hence, promote the most effective use of new data types.

6. Recommendations

Based on the discussion in the previous sections, it is concluded that at the present time, a specific program in gravity field modeling is not only appropriate but it is required. To satisfy the current needs, the participants in the workshop recommend that NASA should:

I. Initiate a program to develop improved models of the gravity field using the existing data supplemented by selected observations from ongoing observing programs. The results from the gravity field improvement program should be available within three to five years. In particular, improved accuracy in the long wave length and short wave length features of the gravity field should be sought with the objective of reducing the contribution to the gravity field error due to these components by 50%. To achieve this objective, the gravity field improvement program should include:

A. Improved utilization of satellite data

- The potentially useful satellite data which should be examined are given by Items 1 through 6 in Paragraph 4A.
- 2. The computation procedures should include:
 - The development and utilization of better physical, statistical and dynamic models,
 - b. The development of improved parameter estimation techniques with cognizance of modern computer technology.
 - c. Development of a standard for comparison and validation of techniques and software, and
 - t. The development of procedures for the use of the computer facilities at the NASA Goddard Space Flight Center by outside investigators.

- B. Improved utilization of surface data and i.s combination with satellite data. This goal would require:
 - The acquisition of more complete surface gravimetry data.
 - Improved anomaly estimates for land areas now covered by values obtained through geophysical prediction procedures.
 - Development of appropriate estimation algorithms for combining satellite and surface gravity measurements.
- C. Improved interaction with users and the gravity field development to define the requirements for:
 - 1. Satellite ephemerides computation.
 - 2. Description of gravity field errors,
 - 3. Support of ocean dynamics, and
 - 4. The development of short wavelength local gravity models for geophysical studies.
- II. In recognition of the challenging analysis effort required to achieve the objectives of a GRAVSAT mission, NASA should support the development of techniques for analysis of data generated by a GRAVSAT mission. In particular, it should:
 - A. Stimulate and support research designed to cope with and take advantage of the GRAVSAT data. This data may require different analysis techniques than those used previously.
 - B. Consider the impact of GRAVSAT requirements in establishing the interim gravity field development program.
- III. To implement these recommendations, it is proposed that NASA:
 - A. Establish a continuing Steering Committee drawn from respondents to an Announcement of Planning Opportunity (APO), supplemented as necessary by selective appointments, to formulate the procedures for developing the interim gravity field models and to assist in the determination of the gravity model.
 - B. The working groups of the steering committee should participate in the data selection and preparation, including data collection (both old and new), data processing,

formation of normal equations and/or other forms of data compression and parameter estimation. Since a significant portion of this effort will be performed at the Goddard Space Flight Center, expansion of current GSFC capabilities to accommodate the program requirements will be necessary.

C. Participation in the data analyses program for the improved interim gravity field should be sought by Announcement of Opportunity for both intra- and extra-agency investigations. Specific arrangements should be made to allow the principal investigators selected under the Announcement of Opportunity the option of conducting this research using the computer facilities at the NASA

Goddard Space Flight Center.

D. The Announcement of Opportunity should also include calls for preparatory work related to the GRAVSAT data analysis.

REFERENCES

- [1] Geodesy: Trends and Prospects, The National Research Council, Committee on Geodesy, 1978.
- [2] Applications of Dedicated Gravitational Satellite Mission, The National Research Council, Committee on Geodesy, Panel on Gravity Field and Sea Level, 1979.
- [3] Report of GRAYSAT Users Working Group, March 12-13, 1980a.
- [4] Report of GRAVSAT Users Working Group, April 24-25, 1980b.
- [5] Interagency Plan for Coordination and Use of Earth Gravity Field
 Survey Data From Space, Satellite Geodesy Applications Board,
 July 1980.
- [6] Satellite Altimetric Measurements of the Ocean, Report of the TOPEX Science Working Group, March 1981.
- [7] Jekeli, C. and R. Rapp, "Accuracy of the Determination of Mean Anomalies and Mean Geoid Undulations from a Satellite Gravity Field Mapping Mission," Dept. of Geodetic Science, Report No. 307, Ohio State Univ., August 1980.